JOSEPH HELSZAJN

MICROWAVE POLARIZERS, POWER DIVIDERS, PHASE SHIFTERS, CIRCULATORS, AND SWITCHES





Microwave Polarizers, Power Dividers, Phase Shifters, Circulators, and Switches

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Joseph Helszajn Heriot Watt University, Edinburgh, UK





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Editorial Office 111 River Street, Hoboken, NJ 07030, USA

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Library of Congress Cataloging-in-Publication Data

Names: Helszajn, J. (Joseph) author.
Title: Microwave polarizers, power dividers, phase shifters, circulators, and switches / authored by Joseph Helszajn.
Description: First edition. | Hoboken, NJ : John Wiley & Sons, Inc., [2019] | Includes bibliographical references and index. |
Identifiers: LCCN 2018027839 (print) | LCCN 2018038875 (ebook) | ISBN 9781119490081 (Adobe PDF) | ISBN 9781119490074 (ePub) | ISBN 9781119490050 (hardcover)
Subjects: LCSH: Microwave devices.
Classification: LCC TK7876 (ebook) | LCC TK7876 .M267 2019 (print) | DDC 621.381/33–dc23 LC record available at https://lccn.loc.gov/2018027839
Cover design: Wiley

Cover image: © iStock.com/Ivanastar

Set in 10/12pt Warnock by SPi Global, Pondicherry, India

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

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Preface

Ferrite phase-shifter and control devices are widely used in conjunction with passive microwave circuits in beam shaping and steering of array antennas and in multichannel switching. The intention of this text is to provide the reader with some preliminary insight into the operation of some basic ferrite control devices and to note some system uses. In the beam steering application, variable phase-shifters are employed to tilt the beam of a simple one-dimensional array or more sophisticated two- and three-dimensional ones. Beam shaping is achieved by using variable power dividers and switches. At modest microwave wavelengths, this is often done with the aid of semiconductor devices, but at very high power levels and at millimeter wavelengths, ferrite devices are used almost exclusively. A drawback of the ferrite control device is its longer switching time; its microwave power rating is, however, usually superior. Although many ferrite devices are nonreciprocal, this is often not essential or indeed desirable in the control area. Mechanically actuated passive switches and variable phase shifters using rotatable half-wave plates are other possibilities. Multichannel switching may consist of making provisions for switching on a standby transmitter in case of a failure mode in some simple radar or satellite equipment or it may involve the control of a high-power signal using Butler matrices; it may also be utilized in the construction of multiport power combiners. The threeport junction circulator is, of course, also ideally suited for switching a signal at one port to any of n - 1 others. Frequency reuse where spatially isolated beams operate in the same frequency band is another area where power dividers and variable phase shifters are required. Switching of the hand of polarization of a wave or rotating its polarization are other applications. Microwave ferrite phase-shifters and other devices essentially rely for their operation on the different birefringences exhibited by a magnetized magnetic insulator under the influence of different direct and alternating magnetic fields. Nonlinear effects or spinwave instabilities at large signal levels are a separate consideration.

Acknowledgments

This text is dedicated to my colleagues and friends without whom this book would not have been possible: Mark McKay, Marco Caplin, Henry Downs, David J. Lynch, John Sharp, William D'Orazio, Alicia Casanueva, and Angel Mediavilla Sánchez.

List of Contributors

Mr. Marco Caplin

RF Designer Apollo Microwaves Ltd Dorval, Quebec Canada

Professor Alicia Casanueva

Communication Engineering Department University of Cantabria Santander Spain

Mr. William D'Orazio

RF Designer Apollo Microwaves Ltd Dorval, Quebec Canada

Mr. Henry Downs

Chief Science Officer – EVP Engineering Mega Industries, LLC Gorham, ME USA

Dr. David J. Lynch

Director Filtronic Wireless Ltd Salisbury, MD USA

Dr. Mark McKay

Principal Engineer Honeywell Edinburgh UK

Professor Angel Mediavilla Sánchez

Communication Engineering Department University of Cantabria Santander Spain

Professor John Sharp

Professor Emeritus Napier University Edinburgh UK

Microwave Switching Using Junction Circulators

1

Joseph Helszajn

Heriot Watt University, Edinburgh, UK

1.1 Microwave Switching Using Circulators

Since the direction of circulation of a circulator is determined by that of the direct magnetic field, it may be employed to switch an input signal at one port to either one or the other two. Switching is achieved by replacing the permanent magnet by an electromagnet or by latching the microwave ferrite resonator directly by embedding a current carrying wire loop within the resonator.

The schematic diagram of a switched junction is shown in Figure 1.1a. It is particularly useful in the construction of Butler-type matrices in phase array systems. A single-pole three throw version is depicted in Figure 1.1b.

Two common arrangements in which ferrite circulators may be employed to obtain microwave switching are separately illustrated in Figure 1.1c and 1.1d. The first uses a circulator in conjunction with a pin diode switch to vary the short-circuit plane terminating port 2. A transmission analog phase shifter is therefore obtained between ports 1 and 3 with this mode of operation. The second version is also a transmission configuration but now a switchable circulator is used to control the path between ports 1 and 3 of the circulator. The switching speed of the pin device is normally the faster one.

1.2 The Operation of the Switched Junction Circulator

The adjustment of a fixed field circulator or a switched circulator is a two-step procedure. The first fixes its midband frequency and the second its gyrotropy. A phenomenological description of these two operations is illustrated in

1







(a)



(d)



Figure 1.1 Microwave phase shifter using (a) schematic of circulator switch, (b) SP4T Butler switch using circulators, (c) pin dioded switch and fixed circulator, and (d) switched circulator.



Figure 1.2 Standing wave patterns in (a) demagnetized stripline junction and (b) magnetized stripline junction.

Figure 1.2a and b in the case of a stripline geometry. The direction of circulation is here fixed by the sense of the direct magnetic field intensity along the axis of the resonator. This may be done by either internally latching the hysteresis loop of the magnetic insulator between its two remanent states or by having recourse to an external magnetic circuit. The electric field pattern may be rotated either clockwise or anticlockwise by splitting the degeneracy of the counterrotating field patterns of the resonator. A latched stripline geometry is indicated in Figure 1.3.

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Figure 1.3 Current and magnetic field in ferrite disc.

1.3 The Turnstile Circulator

The waveguide junction switch is usually but not exclusively based on a Faraday rotation effect along a quarter-wave long cavity resonator open-circuited at one flat face and short-circuited at the other. Its first circulation condition is a 90°

(00⁰

cavity with no rotating of the electric field pattern, which is again a figure of eight pattern. Its second circulation condition is obtained by replacing the dielectric resonator by a gyromagnetic insulator. The effect is to rotate the polarization of the electric field by a 15° angle in the positive direction of propagation and a further 15° in the opposite direction. The total rotation places an electric null at a typical output port.

Figure 1.4a and b are sketches of the electric and magnetic HE_{11} standing wave patterns about midway along the cavity. The electric field is zero at the electric wall of the cavity, whereas the magnetic field is zero at its magnetic flat wall.



3



Figure 1.5 Schematic diagram of externally latched circulator using a post-resonator waveguide junction.



Figure 1.6 Schematic diagram of waveguide junction circulator using a partial height: (a) triangular and (b) circular resonator with a wire loop.

1.4 Externally and Internally Latched Junction Circulators

Circulators may be either actuated by an electromagnet or they may be operated by internally or externally latching the ferrite resonator. Figure 1.5 illustrates one externally latched arrangement. Figure 1.6a and b depict internally latched waveguide devices using half-wave or quarter-wave long resonators.

Figure 1.7 indicates the two possible wire configurations met in the construction of a waveguide switch using a prism resonator.



Figure 1.7 Schematic diagrams of waveguide circulators showing different switching wire configurations.

1.5 Standing Wave Solution of Resonators with Threefold Symmetry

Two resonators met in the design of switched circulators with threefold symmetry are the equilateral triangle structure and the quasi WYE geometry.

Microwave Polarizers, Power Dividers, Phase Shifters, Circulators, and Switches



Figure 1.8 Standing wave solution of three-port circulators using (a) triangular resonator and (b) WYE resonator.

The standing wave solution of the second circulation solutions is here not obvious but each may be constructed by taking suitable linear combinations of those of the first circulation condition. Figure 1.8a and b illustrate the equipotential lines of the standing wave patterns in each situation.

1.6 Magnetic Circuit Using Major Hysteresis Loop

The direct magnetic field in a junction circulator can be established using either an external electromagnet or it can be switched by current pulses through a magnetizing wire between the two remanent states of the major or indeed of a minor hysteresis loop of a closed magnetic circuit. The former arrangement requires a holding current to hold the device in a given state.

In the latter one, however, no such current is necessary; the device remains latched in a given state until another switching operation is required. The advantages and disadvantages of each type of circuit are understood.

Operation on the major hysteresis loop may be understood by scrutinizing the hysteresis loop in Figure 1.9, providing it is recognized that the size and shape of this loop may vary with the speed of the switching process. In this situation, the magnetization of the toroid is driven between two remanent states $(\pm 4\pi M_r)$

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Figure 1.9 Typical hysteresis loop of a latching phase shifter operating with a major hysteresis loop switching.

equidistant from the origin by the application of a current pulse sufficiently large to produce a field perhaps three or five times that of the coercive force.

After this point is reached, the current pulse is removed and the magnetization will move to the remanent value $(\pm 4\pi M_r)$ and remain there until another switching operation is desired. This sort of electronic driver circuit is relatively simple since it is only required that the toroids be driven back and forth between the major remanent states of the hysteresis loop.

1.7 Display of Hysteresis Loop

The magnetic properties and parameters of a magnetic core or toroid under different operating conditions, such as temperature, say, are best discussed in terms of the details of its hysteresis loop.

Some experimental quantities that are of particular interest include the saturation magnetization (M_0), the remanent magnetization (M_r), and the coercive force (H_c). The experimental display of such loops is therefore of some interest. One circuit that may be used for this purpose is outlined in Figure 1.10. This



Figure 1.10 Schematic diagram of hysteresis display.

arrangement develops voltage $V_{\rm p}$ and $V_{\rm i}$ that are proportional to *B* and *H*, respectively.

The magnetic field (*H*) in the core is monitored by measuring the voltage (V_p) across a resistor in series with the primary winding, see Figure 1.10.

$$H = \frac{N_{\rm p}}{I_{\rm p}} \left(\frac{V_{\rm p}}{R_{\rm p}} \right), \text{A m}^{-1}$$
(1.1)

where I_p is the effective of the primary winding, N_p is the number of turns of the primary winding (10–30), and R_p is the resistor in series with the primary coil (10 Ω). The magnetization (*B*) is likewise evaluated by forming the voltage (V_i) across the capacitance of the RC integrator in the secondary circuit.

$$B \approx \frac{-V_i R_i C_i}{N_s A} \tag{1.2}$$

where R_i is the series resistance of the integrator (100 k Ω), C_i is the capacitance of the integrator (0.10 μ F), N_s is the number of turns of the secondary winding (10–30), and A is the cross-sectional area of the core.

The data shown in Figure 1.11 on the effects of small air gaps on the squareness of the hysteresis loop have been obtained using the arrangement outlined here.